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APPLICATIONS OF INFRARED BAND MODEL CORRELATIONS TO NONGRAY RADIATION

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Abstract—Various correlations for the total absorptance of a wide band are presented. These are employed in two physically realistic problems (radiative transfer in gases with internal heat source and heat transfer in laminar flow of absorbing–emitting gases between parallel plates) to study their influence on final radiative transfer results.

NOMENCLATURE

A, band absorptance [cm^{-1}];
$A(u, \beta)$, band absorptance of a wide band [cm^{-1}];
$A_N(u, \beta)$, band absorptance of a narrow band [cm^{-1}];
A_0, band width parameter [cm^{-1}];
C_0, correlation parameter [$atm^{-1}-cm^{-1}$];
e_ω, Planck's function [$(W\,cm^{-2})/cm^{-1}$];
e_{ω_0}, Planck's function evaluated at wave number ω_0;
$E_n(x)$, exponential integral of order n;
H, gas property for the large path length limit;
L, distance between plates;
M, radiation–conduction interaction parameter for the large path length limit;
N, optically thin radiation–conduction interaction parameter;
P, pressure [atm];
q_R, total radiation heat flux [W/cm^2];
q_{R_ω}, spectral radiation heat flux [$(W\,cm^{-2})/cm^{-1}$];
q_w, wall heat flux [W/cm^2];
Q, heat source or sink [W/cm^3];
S, integrated intensity of a wide band [$atm^{-1}\,cm^{-2}$];
S_j, intensity of the jth spectral line;
t, line structure parameter $= \beta/2 = \pi\gamma_L/d$;
T, equilibrium temperature [K];
T_1, T_2, wall temperature [K];
T_b, bulk temperature [K];
u, dimensionless coordinate $= SX/A_0$;
u_0, dimensionless path length $= SPL/A_0$;
X, pressure path length $= py$;
y, transverse coordinate [cm];
β, line structure parameter $= 2t = 2\pi\gamma_L/d$;
γ_j, line half-width [cm^{-1}];
θ, dimensionless temperature defined in equation (18);
θ_b, dimensionless bulk temperature;
κ_ω, equilibrium spectral absorption coefficient [cm^{-1}];
λ, thermal conductivity [$(W\,cm^{-2})/K$];
ξ, dimensionless coordinate $= y/L = u/u_0$;
ϕ, dimensionless function defined in equation (17);
ω, wave number [cm^{-1}];
ω_0, wave number at the band center [cm^{-1}].

1. INTRODUCTION

THE TOTAL absorption of a band of overlapping lines strongly depends upon the line intensity, the line half-width, and the spacing between the lines. In a particular band, the absorption coefficient varies very rapidly with the frequency and, therefore, it becomes a very difficult and time-consuming task to evaluate the total band absorptance by numerical integration over the actual band contour. Consequently, several approximate band models (narrow as well as wide) have been proposed [1–14] which represent absorption from an actual band with reasonable accuracy. Several continuous correlations for the total band absorptance are available in the literature [11–15]. These are employed here in two physically realistic radiative transfer analyses to study their effects on the final results of actual radiative transfer process. The first problem considered is the problem of radiative energy transfer between a gas volume and the surrounding walls. The physical concept of this problem can be applied to the radiative transfer analyses of plane-parallel atmosphere with non-uniform heating. The second problem considered is that of a laminar flow of an absorbing–emitting gas between parallel surfaces. Attention is directed, in particular, to carbon monoxide and carbon dioxide gases. Various correlations for the total band absorptance are presented in Section 2, and these are employed in radiative transfer analyses in Section 3.

2. BAND ABSORPTANCE AND CORRELATIONS

The absorption within a narrow spectral interval of a vibration rotation band can quite accurately be represented by the so-called "narrow band models". For a homogeneous path, the total absorptance of a narrow band is given by

$$A_N = \int_{\Delta\omega} [1-\exp(\kappa_\omega X)]\,d\omega \qquad (1)$$

where κ_ω is the volumetric absorption coefficient, ω is the wave number, and $X = py$ is the pressure path length. The limits of integration in equation (1) are over the narrow band pass considered. The total band absorptance of the so-called "wide band models" is given by

$$A = \int_{-\infty}^{\infty} [1-\exp(-\kappa_\omega X)]\,d(\omega-\omega_0) \qquad (2)$$

where the limits of integration are over the entire band pass and ω_0 is the wave number at the center of the wide band. In actual radiative transfer analyses, the quantity of frequent interest is the derivative of equations (1) and (2).

Four commonly used narrow band models are Elsasser, Statistical, Random Elsasser, and Quasi-Random. The application of a model to a particular case depends upon the nature of the absorbing–emitting molecule. Complete discussions on narrow band models, and expressions for transmittance and integrated absorptance are available in the literature [1–6, 13, 14]. Detailed discussions on the wide band models are given in [7–15]. The relations for total band absorptance of a wide band are obtained from the absorptance formulations of narrow band models by employing the relation for the variation of line intensity as [10, 13–15]

$$S_j/d = (S/A_0)\exp\{[-b_0|\omega-\omega_0|]/A_0\} \qquad (3)$$

where S_j is the intensity of the jth spectral line, d is the line spacing, S is the integrated intensity of a wide band, A_0 is the band width parameter, and $b_0 = 2$ for a symmetrical band and $b_0 = 1$ for bands with upper and lower wave number heads at ω_0. The total absorptance of an exponential wide band, in turn, may be expressed by

$$\bar{A}(u,\beta) \equiv A(u,\beta)/A_0 = \frac{1}{A_0}\int_{\substack{\text{wide}\\ \text{band}}} [A_N(u,\beta)]\,d(\omega-\omega_0) \qquad (4)$$

where $u = SX/A_0$ is the nondimensional path length, $\beta = 2\pi\gamma_L/d$ is the line structure parameter, γ_L is the Lorentz line half-width, and $\bar{A}_N(u,\beta)$ represents the mean absorptance of a narrow band.

By employing the Elsasser narrow band absorptance relation and equation (3), the expression for the exponential wide band absorptance is obtained as [13, 14]

$$\bar{A}(u,\beta) = \gamma + (1/\pi)\int_0^\pi [\ln\psi + E_1(\psi)]\,dz \qquad (5)$$

where $\psi = u\sinh\beta/(\cosh\beta-\cos z)$, $\gamma = 0.5772156$ is the Euler's constant, and $E_1(\psi)$ is the exponential integral of the first order. Analytic solution of equation (5) can be obtained in a series form as [14]

$$\bar{A}(u,\beta) = \sum_{n=1}^{\infty}\{-(A)^n[SUM(mn)]/[n(B+1)^n n!(n-1)!]\} \qquad (6)$$

where

$$SUM(mn) = \sum_{m=0}^{\infty}[(n+m-1)!(2m-1)!C^m]/[2^m(m!)^2]$$

$$A = -u\tanh\beta,\quad B = 1/\cosh\beta,$$

$$C = 2/(1+\cosh\beta) = 2B/(B+1).$$

The series in equation (6) converges rapidly. When the weak line approximation for the Elsasser model is valid (i.e. β is large), then equation (5) reduces to [13, 14]

$$\bar{A}(u) = \gamma + \ln(u) + E_1(u). \qquad (7)$$

In the linear limit, equations (5) and (6) reduce to $\bar{A} = u$, and in the logarithmic limit they reduce to $\bar{A} = \gamma + \ln(u)$. It can be shown that equation (5) reduces to the correct limiting form in the square-root limit [14]. Results of equations (5) and (6) are found to be identical for all pressures and path lengths. For $p > 1$ atm, results of equations (5)–(7) are in good agreement for all path lengths.

By employing the uniform statistical, general statistical, and random Elsasser narrow band models absorptance relations and equation (3), three additional expressions for the exponential wide band absorptance were obtained in [13, 14]. The absorptance results of the four wide band models are discussed in detail in [14]. The expression obtained by employing the uniform statistical model also reduces to the relation (7) for large β.

Several continuous correlations for the total absorptance of a wide band, which are valid over different values of path length and line structure parameter, are available in the literature. These are discussed, in detail, in [11–15] and are presented here in the sequence that they became available in the literature. Most of these correlations are developed to satisfy at least some of the limiting conditions (nonoverlapping line, linear, weak line, and strong line approximations, and square-root, large pressure, and large path length limits) for the total band absorptance [12, 14]. Some of the correlations even have experimental justifications [8, 11].

The first correlation for the exponential wide band absorptance (a three piece correlation) was proposed by Edwards *et al.* [7, 8]. The first continuous correlation was proposed by Tien and Lowder [11], and this is of the form

$$\bar{A}(u,\beta) = \ln(uf(t)\{(u+2)/[u+2f(t)]\}+1) \qquad (8)$$

where

$$f(t) = 2.94[1-\exp(-2.60t)],\quad t = \beta/2.$$

This correlation does not reduce to the correct limiting form in the square-root limit [12], and its use should be made for $\beta \geqslant 0.1$. Another continuous correlation was proposed by Goody and Belton [16], and in terms of the present nomenclature, this is given by

$$\bar{A}(u,\beta) = 2\ln\{1+u/[4+(\pi u/4t)]^{1/2}\},\quad \beta = 2t. \qquad (9)$$

Use of this correlation is restricted to relatively small β values [12–14]. Tien and Ling [17] have proposed a simple two parameter correlation for $\bar{A}(u,\beta)$ as

$$\bar{A}(u) = \sinh^{-1}(u) \qquad (10)$$

which is valid only for the limit of large β. A relatively simple continuous correlation was introduced by Cess and Tiwari [12], and this is of the form

$$\bar{A}(u,\beta) = 2\ln\left(1+u/\{2+[u(1+1/\bar{\beta})]^{1/2}\}\right) \quad (11)$$

where $\bar{\beta} = 4t/\pi = 2\beta/\pi$. By slightly modifying equation (11), another form of the wide band absorptance is obtained as [13, 14]

$$\bar{A}(u,\beta) = 2\ln\left(1+u/\{2+[u(c+\pi/2\beta)]^{1/2}\}\right) \quad (12)$$

where

$$c = \begin{cases} 0.1, & \beta \leqslant 1 \text{ and all } u \text{ values} \\ 0.1, & \beta > 1 \text{ and } u \leqslant 1 \\ 0.25, & \beta > 1 \text{ and } u > 1. \end{cases}$$

Equations (11) and (12) reduce to all the limiting forms [12]. Based on the formulations of slab band absorptance, Edwards and Balakrishnan [10] have proposed the correlation

$$\bar{A}(u) = \ln(u)+E_1(u)+\gamma+\tfrac{1}{2}-E_3(u) \quad (13)$$

which is valid for large β. For present application, this correlation should be modified by using the technique discussed in [13, 14]. Based upon the formulation of the total band absorptance from the general statistical model, Felske and Tien [15] have proposed a continuous correlation for $\bar{A}(u,\beta)$ as

$$\bar{A}(u,\beta) = 2E_1(t\rho_u)+E_1(\rho_u/2)-E_1[(\rho_u/2)(1+2t)] + \ln[(t\rho_u)^2/(1+2t)]+2\gamma \quad (14)$$

where

$$\rho_u = \{(t/u)[1+(t/u)]\}^{-1/2}.$$

The absorptance relation given by equation (7) is another simple correlation which is valid for all path lengths and for $t = (\beta/2) \geqslant 1$. The relation of equation (6) can be treated as another correlation applicable to gases whose spectral behavior can be described by the Elsasser model. In [14] Tiwari and Batki have shown that the Elsasser as well as random band model formulations for the total band absorptance reduce to equation (7) for $t \geqslant 1$.

Band absorptance results of various correlations are compared and discussed in some detail in [13, 14, 18]. It was found that results of these correlations could be in error by as much as 40% when compared with the exact solutions based on different band models. Felske and Tien's correlation was found to give the least error when compared with the exact solution based on the general statistical model while Tien and Lowder's correlation gave the least error when compared with the exact solution based on the Elsasser model. The results of Cess and Tiwari's correlations followed the trend of general statistical model. Tiwari and Batki's correlation [equation (6) or (7)] was found to provide a uniformly better approximation for the total band absorptance at relatively high pressures. The sole motivation in presenting the various correlations here is to see if their use in actual radiative processes made any significant difference in the final results.

3. RADIATIVE TRANSFER ANALYSES

The band absorptance correlations discussed in the previous section are employed in this section to two illustrative radiative transfer problems. The physical model and coordinate systems for both problems are shown in Fig. 1.

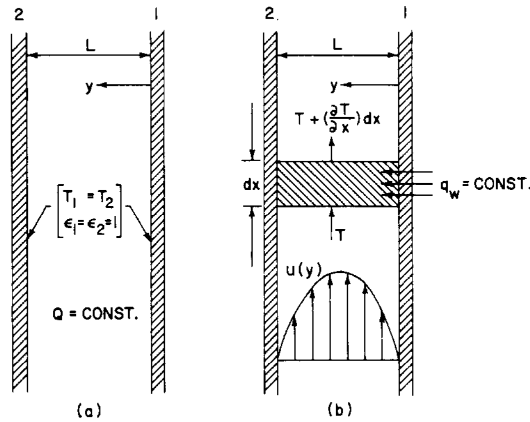

FIG. 1. Physical model and coordinate system.

The expression for the total radiative flux, in general, may be given by [12, 19]

$$q_R(\xi) = e_1 - e_2 + \tfrac{3}{2}A_0u_0 \times \left\{ \int_0^{\xi} [e_{\omega 0}(\xi') - e_{\omega_0}(T_1)]\bar{A}'[\tfrac{3}{2}u_0(\xi-\xi')]\,\mathrm{d}\xi' - \int_{\xi}^{1} [e_{\omega 0}(\xi') - e_{\omega 0}(T_2)]\bar{A}'[\tfrac{3}{2}u_0(\xi'-\xi)]\,\mathrm{d}\xi' \right\} \quad (15)$$

where

$$\xi = y/L,\ u_0 = (S(T_1)pL/A_0(T_1),\ \bar{A}(u,\beta) = A(u,\beta)/A_0.$$

In this equation $e = \sigma T^4$, with σ denoting the Stefan–Boltzmann constant, $\bar{A}'(u)$ denotes the derivative of $\bar{A}(u)$ with respect to u, and u_0 represents the non-dimensional pressure path length. The derivation of equation (15) assumes the existence of a local thermodynamic equilibrium and employs the exponential kernal approximation. The Planck function, $e_\omega(T)$, is considered a slowly varying function of wave number over the band and its value is evaluated at the band center. Furthermore, small temperature differences within the gas were assumed and the absorption coefficient (like any other physical property) was taken to be independent of temperature [12]. Equation (15) possesses two limiting forms (optically thin and large path length limits) which are discussed in detail in [12, 20, 21]. It should be pointed out here that while the large path length limit may depend upon a particular band model employed in the radiative flux equation, the optically thin limit is completely independent of the band model.

In order to study the effect of different band absorptance correlations on the final results of actual radiative transfer processes, two illustrative cases (extensively studied in the literature) are treated here. The first is the problem of radiative energy transfer between a gas volume (where there is a uniform heat source) and the surrounding walls. The physical concept of this problem can be applied to the radiative transfer analyses of plane-parallel atmosphere. The second problem considered is the problem of laminar flow of absorbing emitting gases between parallel surfaces. This represents a realistic situation of convective–radiative energy transfer in the atmosphere and in other engineering problems. For mathematical simplicity, the bounding surfaces for both problems are considered to be black.

3.1. *Radiative transfer in gases with internal heat source*

The physical model and the coordinate system for this case is shown in Fig. 1(a). The two parallel black walls are considered to be at the same constant temperature T_1. The gas is assumed to have a uniform heat source (or sink) of strength Q per unit volume. The radiative transfer is the sole mechanism of energy transfer within the gas. The local temperature distribution is thus a consequence of the uniform heat source adding energy to the gas which in turn is transferred through the gas to the bounding surfaces by radiative transfer.

For this special case, the conservation of energy yields

$$\mathrm{d}q_R/\mathrm{d}y = Q. \tag{16}$$

Since the problem is symmetric, an integration of equation (16) gives $q_R = (1/2)QL(2\xi - 1)$ and after combining this with equation (15) there is obtained

$$\xi - \tfrac{1}{2} = \tfrac{3}{2}\left\{\int_0^{\xi} \phi(\xi')\bar{A}'[\tfrac{3}{2}u_0(\xi-\xi')]\,\mathrm{d}\xi' - \int_{\xi}^{1} \phi(\xi')\bar{A}'[\tfrac{3}{2}u_0(\xi'-\xi)]\,\mathrm{d}\xi'\right\} \tag{17}$$

where

$$\phi(\xi) = [e_{\omega_0}(T) - e_{\omega_0}(T_1)]/(Q/PS(T_1)).$$

In this equation $\phi(\xi)$ represents the temperature profile within the gas in terms of the Planck function. The equation is written for a single band gas but it can easily be extended to the case involving multi-band gases [12].

It has been shown in [12, 20, 21] that equation (17) gives the result $\phi = 1/3$ in the optically thin limit while it yields $(\phi/u_0) = (1/\pi)[\xi(1-\xi)]^{1/2}$ in the large path length limit. By employing the various band absorptance correlations discussed in the previous section, numerical solutions of equation (17) were obtained for different values of the line structure parameter $t = \beta/2$. The results (i.e. centerline temperatures) are illustrated in Figs. 2–4 along with the limiting solutions. It should be pointed out here that these results apply to any situation for which radiative transfer within the gas is the result of a single band.

It is obvious from Figs. 2–4 that the band absorptance results of all the correlations approach the optically thin limit for small u_0, the influence of the line structure parameter is maximum for intermediate values of u_0, and in the large path length limit the solutions become independent of β. The one exception to this is that the results of Goody and Belton's correlation do not approach the correct logarithmic limit for large β. The reasons for this are discussed in [12, 14] where it was pointed out that the use of the Goody and Belton's correlation should be restricted to relatively small values of β.

For a particular value of β, the results of different correlations approach the linear and logarithmic limits for different u_0 values. For $\beta = 0.02$, for example, the results of all correlations (except Tien and Lowder's) almost are identical for $u_0 \geqslant 1$. This corresponds to the range of the square-root limit where three separate conditions ($\beta \ll 1$, $u/\beta \gg 1$, and $\beta \ll 1$) must be satisfied [12, 14]. The square-root limit is not satisfied by the Tien and Lowder's correlation. Although the linear limit is independent of any spectral model, it is approached by different correlations for different u_0 values.

Maximum differences between the results of various correlations occur for the intermediate values of u_0 ($0.1 < u_0 < 10$) and for $0.1 < \beta < 1$. For large β values (i.e. for high pressures), the line structure of a gas is smeared out and differences between the results of various correlations become small (see Fig. 4). This situation corresponds to the weak line approximation [14], and some consideration of this usually is given in developing a particular correlation.

In general, the temperature difference between the centerline of the physical system and the wall is found to be lowest for the Tien and Lowder's correlation and highest for the Cess and Tiwari's correlations. For the most part, the results of other correlations fall between these two extreme values. The physical reasoning behind this is consistent with the results of the total band absorptance presented in [13, 14, 18]. If the absorption by a gas is high, then, to maintain the local thermodynamic equilibrium, the emission by the gas will be higher. This will result in high radiative energy transfer from the gas and consequently will lower the centerline temperature. Thus, the difference in the centerline temperatures, as obtained by using the different correlations, are of the same order as the difference in the band absorptance results of various correlations.

As discussed in [4, 5, 14], the Elsasser theory always predicts higher absorption than the general statistical model. From this and the results of Figs. 2–4, it may be concluded that for $\beta > 0.02$, use of the Tien and Lowder's correlation is justified for gases whose spectral behavior can be described by the regular Elsasser model. On the other hand, use of the Cess and Tiwari's correlation could be made to gases with bands of highly overlapping lines. Since, in a vibration–rotation band, the lines are distributed neither regularly nor randomly, care should be taken in applying a particular correlation for a specific case.

3.2. *Heat transfer in laminar flow of absorbing–emitting gases*

The second problem considered is the problem of heat transfer in laminar, incompressible, constant

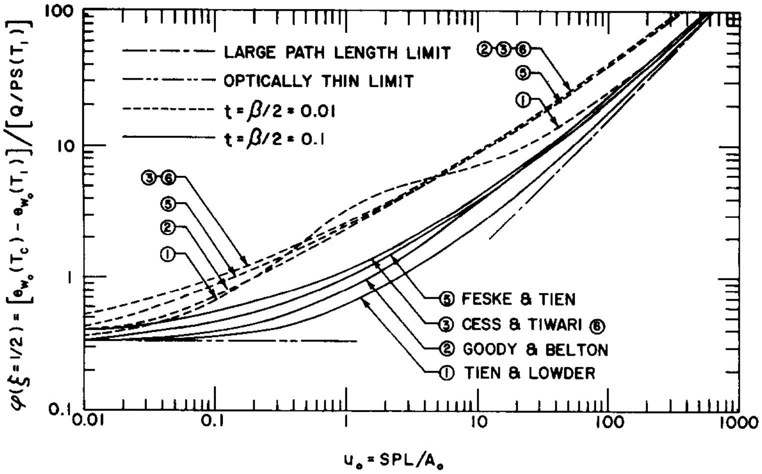

FIG. 2. Comparison of results for a single band gas as obtained by using the various band absorptance correlations.

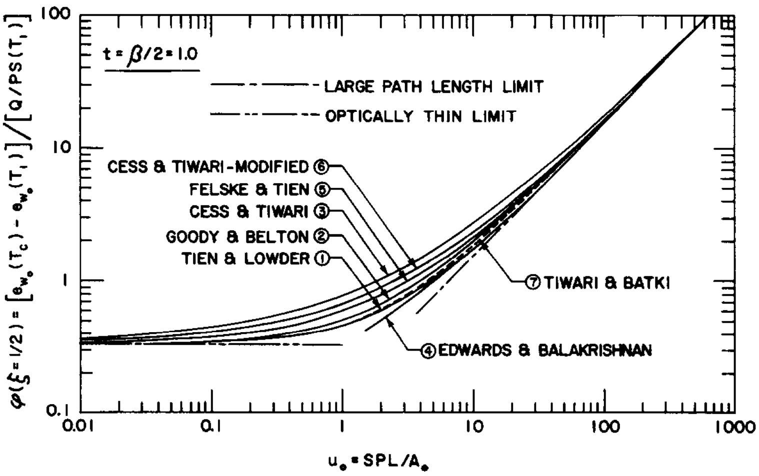

FIG. 3. Comparison of results for a single band gas obtained by using the various band absorptance correlations for $t = 1$.

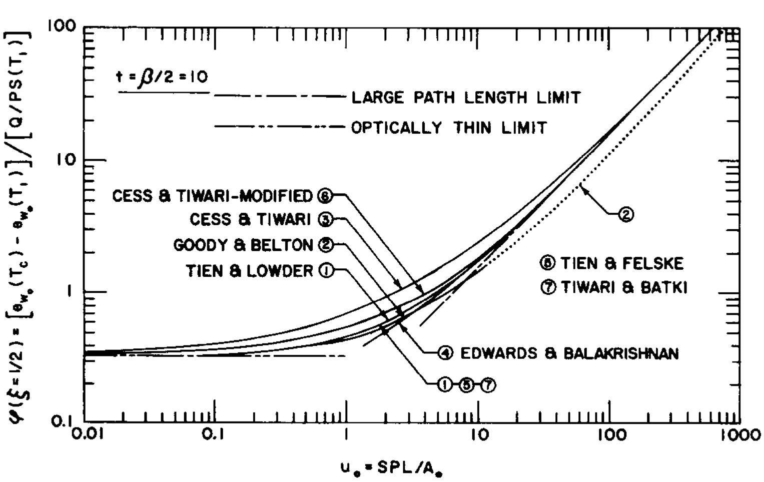

FIG. 4. Comparison of results for a single band gas obtained by using the various band absorptance correlations for $t = 10$.

properties, fully developed flow of absorbing emitting gases between parallel plates. The physical model and coordinate system for this case is shown in Fig. 1(b). The conditions of uniform surface heat flux for each plate is assumed such that the temperature of the plates, T_1, varies in the axial direction. Fully developed heat transfer is considered, and axial conduction and radiation is assumed to be negligible as compared with the normal components. Consistent with the constant properties flow, the absorption coefficient is taken to be independent of temperature and radiation can be linearized. Extensive treatment of this problem is available in the literature [8, 12, 22–27]. The sole motivation for studying the problem here is to investigate the influence of different band absorptance correlations on the radiative transfer capability of a gas in a more realistic situation.

Within the confines of foregoing assumptions, the energy equation for the present problem can be written as [12, 18, 23]

$$\frac{d\theta}{d\xi}-2(3\xi^2-2\xi^3)+1 = \frac{3}{2}\frac{L}{\lambda}\sum_{i=1}^{n}H_i u_{0i}\left\{\int_0^{\xi}\theta(\xi')\bar{A}'[\tfrac{3}{2}u_{0i}(\xi-\xi')]\,d\xi' - \int_{\xi}^{1}\theta(\xi')\bar{A}_i[\tfrac{3}{2}u_{0i}(\xi'-\xi)]\,d\xi'\right\} \quad (18)$$

where

$$\xi = y/L,\quad \theta = (T-T_1)/(q_w L/\lambda)$$
$$\bar{A}_i = A_i/A_{0i},\quad u_{0i} = S_i(T_1)pL/A_{0i}(T_1)$$
$$H_i = A_{0i}(de_{\omega i}/dT)_{T_1},\quad H = \sum_{i=1}^{n} H_i.$$

In this equation $S_i(T_1)$ is the band intensity and $A_{0i}(T_1)$ is the band width parameter for the ith band. The equation describes the temperature profile within the gas for which the boundary condition can be written as $\theta(0) = 0$.

For flow problems, the quantity of primary interest is the bulk temperature of the gas, which may be expressed as [18, 23]

$$\theta_b = (T_b - T_1)/(q_w L/\lambda) = 6\int_0^1 \theta(\xi)(\xi-\xi^2)\,d\xi. \quad (19)$$

The heat transfer q_w is given by the expression, $q_w = h_c(T_1 - T_b)$, where h_c is referred to as the heat-transfer coefficient (W/cm^2 K). In general, the heat-transfer results are expressed in terms of the Nusselt number Nu, a dimensionless quantity defined as, $Nu = h_c D_h/\lambda$. Here, D_h represents the hydraulic diameter, and for parallel plate geometry it equals twice the plates separation, i.e. $D_h = 2L$. Upon eliminating the convective heat-transfer coefficient, h_c, from the expressions for q_w and Nu, a relation between the Nusselt number and the bulk temperature is obtained as

$$Nu = 2Lq_w/\lambda(T_1 - T_b) = -2/\theta_b. \quad (20)$$

The heat-transfer results, therefore, can be obtained either in terms of Nu or θ_b. The general solution of equation (18) is obtained by numerical procedures. Before presenting these results, it would be convenient to obtain the limiting solutions of the general equation.

For the case of negligible radiation, the right hand side of equation (18) becomes zero, and by combining the resulting equation with (19), there is obtained

$$-\theta_b = 17/70. \quad (21)$$

In the optically thin limit, equation (18) reduces to a second order ordinary differential equation from which the expression for the bulk temperature is found to be [18, 23]

$$\theta_b = [1/(3N)^3]\{(576)(3N)^{1/2}(NEXP) - 21.6N^2+72N-288\} \quad (22)$$

where

$$NEXP = \{1-\exp[-(3N)^{1/2}]\}/\{1+\exp[-(3N)^{1/2}]\}$$
$$N = (pL^2/\lambda)\sum_{i=1}^{n} S_i(T_1)(de_{\omega i}/dT)_{T_1}.$$

The quantity N in this equation is referred to as the optically thin radiation–conduction interaction parameter. The results of this equation (θ_b vs N) are illustrated in Fig. 5 along with other limiting solutions.

In the large path length limit, equation (18) reduces to

$$\frac{d\theta}{d\xi}-2(3\xi^2-2\xi^3)+1 = M\int_0^1 \theta(\xi')\,d\xi'/(\xi-\xi') \quad (23)$$

where

$$M = HL/\lambda = (L/\lambda)\sum_{i=1}^{n} H_i = (L/\lambda)\sum_{i=1}^{n} A_{0i}(de_{\omega i}/dT)_{T_1}.$$

The dimensionless parameter M constitutes the radiation–conduction interaction parameter for the large path length limit. Equation (23) does not appear to possess a closed form solution. A numerical solution has thus been obtained, and the results for bulk temperature (θ_b vs M) are illustrated in Fig. 5.

By employing the spectral information given in [8, 11], numerical solutions of equation (18) were obtained for CO, CO_2, H_2O, and CH_4. For these gases, thermal conductivity values were obtained from Tsderberg [28]. Bulk temperature results for CO and CO_2 are presented here in terms of the dimensional quantities L and p. Results for H_2O and CH_4 are given in [18]. Bulk temperature results for CO (fundamental band) and CO_2 (15μ, 4.3μ, and 2.7μ bands) as obtained by employing the various correlations for band absorptance, are illustrated in Figs. 6–9.

Results for CO are illustrated in Figs. 6 and 7 for wall temperatures of 500 and 1000 K respectively. It is evident from these figures that, except for the results of Tien and Lowder's correlation, results of other correlations differ from each other by less than 6% for all pressures and path lengths. For $P = 0.1$ and 1 atm, results differ by not more than 3%. The largest difference of about 6% occurs for $P = 10.0$ atm and $T_1 = 1000$ K. From a close observation of all the results presented in Figs. 6 and 7, it may be concluded that, for low to moderate pressures (say up to 5 atm), any

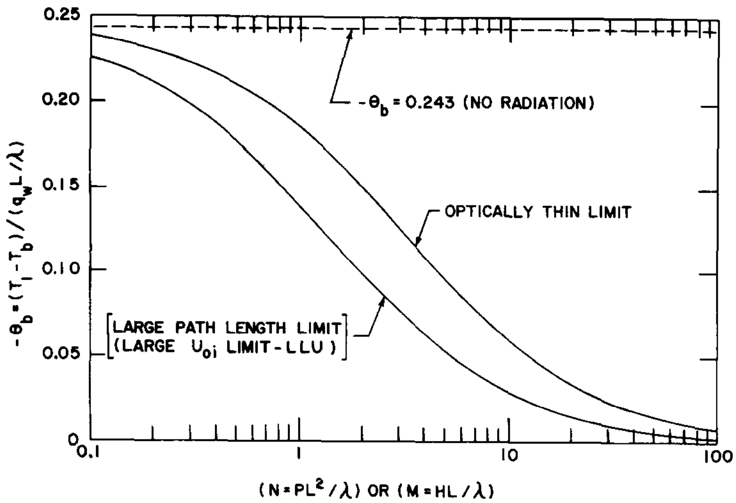

FIG. 5. Limiting solutions of the flow problem. The abscissa for optically thin limit is N and for large path length is M.

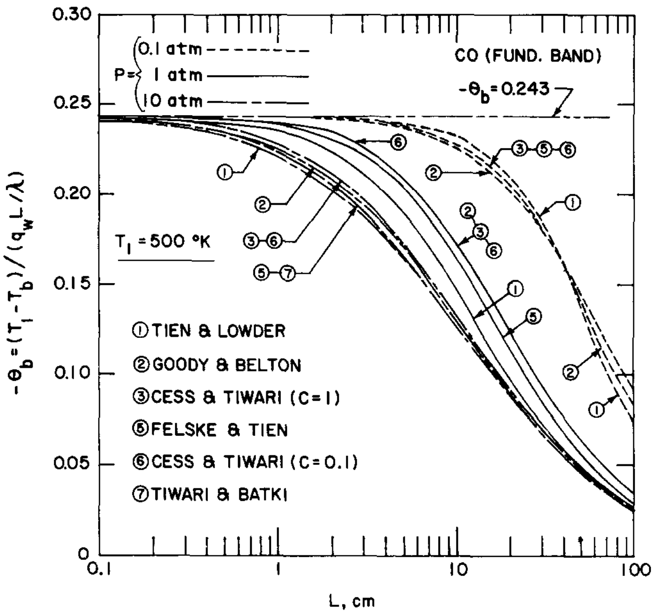

FIG. 6. Results for CO (fundamental band) with $T_1 = 500$ K.

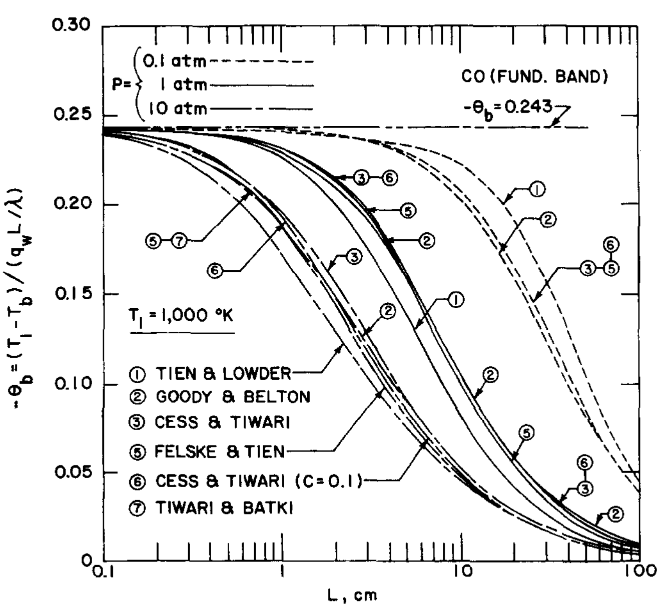

FIG. 7. Results for CO (fundamental band) with $T_1 = 1000$ K.

one of the correlations (Nos. 2, 3, 5, or 6) could be employed in radiative transfer analyses. At high pressures, however, use of correlations 5 or 7 is recommended. In [18, 23], it was noted that for CO the limit of large u_0 is approached at $P = 10$ atm for $T_1 = 500$ K and at 100 atm for $T_1 = 1000$ K. This trend fundamental band having uniform distribution of spectral lines) at moderate and high pressures.

For CO_2, results of different correlations are illustrated in Figs. 8 and 9 for $P = 0.01$, 0.1, 1 and 10 atm, and for $T_1 = 500$ K and 1000 K respectively. In [18], it was noted that for CO_2 the limit of large u_0 (*LLU*)

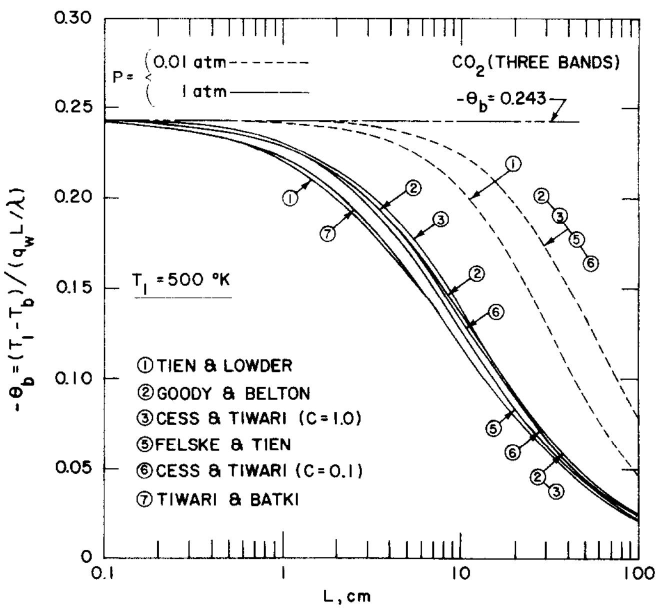

FIG. 8(a). Results for CO_2 (three bands) with $T_1 = 500$ K.

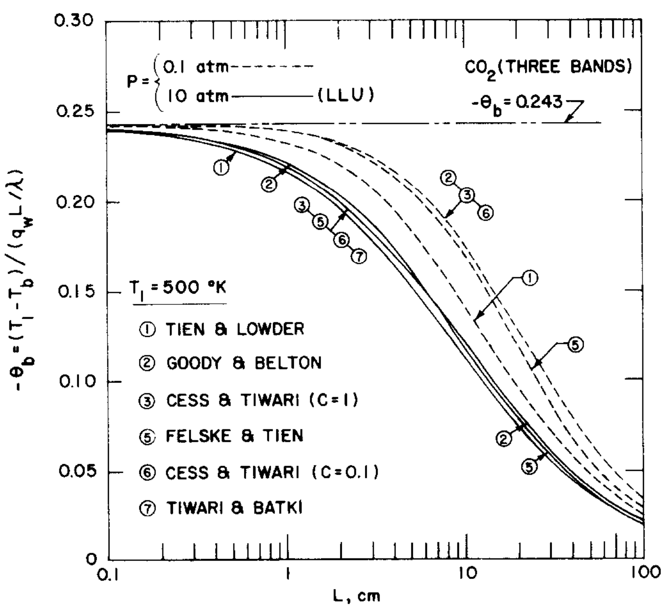

FIG. 8(b). Results for CO_2 (three bands) with $T_1 = 500$ K.

is also evident, in general, from the results of Figs. 6 and 7. The results of Tien and Lowder's correlation, however, follow this trend more closely than any other result. As such, use of the Tien and Lowder's correlation is justified for radiative transfer analyses involving gases like CO (i.e. diatomic gases with single is approached at 2 atm for $T_1 = 300$ K, at about 4 atm for $T_1 = 500$ K, and at about 10 atm for $T_1 = 1000$ K. Thus, results for 10 atm in Figs. 8 and 9 essentially are *LLU* results. For clarity, results of $P = 1$ and 10 atm are not plotted on the same graph.

As was the case with CO, the results of all correlations (except Tien and Lowder's) almost are identical for CO_2 also for $P = 0.01$ and 0.1 atm. This, however, would be expected because the low pressure (small β) situation corresponds to the case of square-root limit and most correlations are developed to satisfy this limit. It was pointed out earlier and in [12, 14] that ferent correlations is about 6% for $P = 1$ atm and $T_l = 1000$ K. For the most part, results of correlations 3, 5, 6 and 7 are identical for $P = 10$ atm. This again would be expected because for CO_2, the *LLU* is approached at relatively lower pressures and most correlations are developed to satisfy the logarithmic

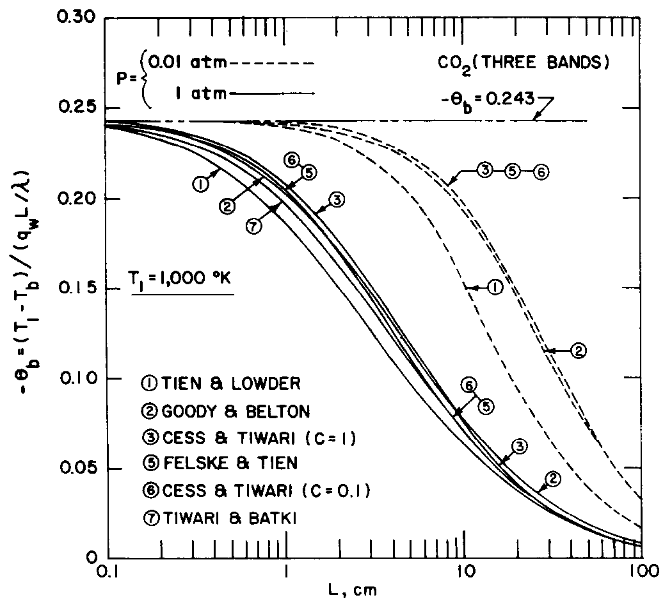

FIG. 9(a). Results for CO_2 (three bands) with $T_l = 1000$ K.

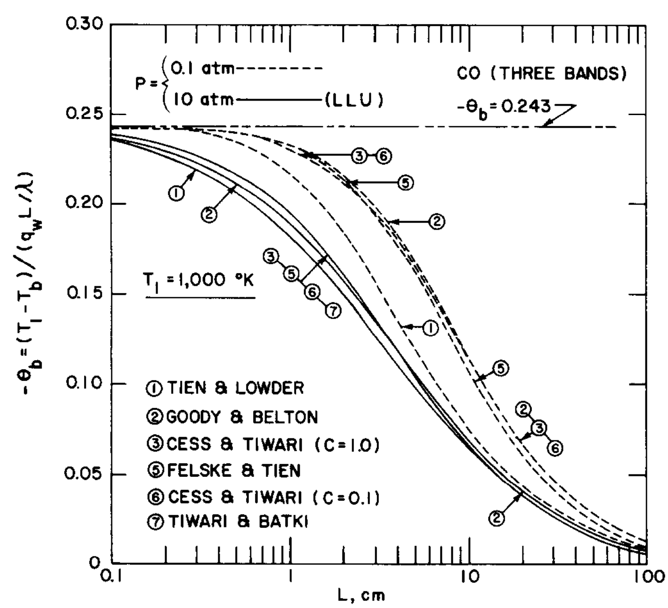

FIG. 9(b). Results for CO_2 (three bands) with $T_l = 1000$ K.

the square-root limit is not satisfied by the Tien and Lowder's correlation. At low pressures, therefore, use of the Tien and Lowder's correlation is not justified. Other results of CO_2 (shown in Figs. 8 and 9) follow the same general trend as for CO in Figs. 6 and 7. The maximum difference between the results of dif- limit. For gases like CO_2, therefore, use of any one of the correlations 2, 3, 5 and 6 is recommended at low and moderate pressures, and of 3, 5, 6 and 7 at high pressures. Use of the correlations 2, 3, 6 and 7, in a particular radiative transfer analysis, provides a greater mathematical flexibility and simplicity.

4. CONCLUSIONS

In this study, use of several continuous correlations for total band absorptance were made to two problems to investigate their influence on the final results of actual radiative processes. For the case of radiative transfer in a gas with internal heat source, it was found that actual centerline temperature results obtained by using the different correlations follow the same general trend as the results of total band absorptance by these correlations. From these results, it may be concluded that use of the Tien and Lowder's correlation should be avoided at lower pressures, but its use is justified (at moderate and high pressures) to gases whose spectral behavior can be described by the regular Elsasser band model. For all pressures and path length conditions, use of the Cess and Tiwari's correlations could be made to gases with bands of highly overlapping lines. In a more realistic problem involving flow of an absorbing–emitting gas, results of different correlations (except the Tien and Lowder's correlation) differ from each other by less than 6% for all pressures and path lengths. Use of Tien and Lowder's correlations is justified for gases like CO at moderate and high pressures. For gases like CO_2, use of any other correlation is recommended. While Felske and Tien's correlation is useful for all pressures and path lengths to gases having random band structure, Tiwari and Batki's simple correlation could be employed to gases with regular or random band structure but for $P \geqslant 1.0$ atm.

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APPLICATION DU MODELE DE BANDES INFRAROUGES AU RAYONNEMENT NON-GRIS

Résumé—Des expressions différentes sont présentées pour l'absorptance totale d'une bande large. Elles sont utilisées dans deux problèmes physiquement réalistes (transfert radiatif dans des gaz avec source de chaleur interne et transfert thermique pour un écoulement laminaire de gaz absorbant et émetteur entre plaques parallèles) et on étudie leur influence sur les résultats du calcul du transfert par rayonnement.

ZUR ANWENDUNG DER KORRELATIONEN FÜR DAS INFRAROTBAND-MODELL AUD FIE NICHTGRAUE STRAHLUNG

Zusammenfassung—Es werden verschiedene Korrelationen für die Gesamtabsorption eines breiten Spektrums angegeben. Um deren Einfluß auf die Ergebnisse von Strahlungsaustauschrechnungen zu erfassen, wurden die Korrelationen auf zwei physikalisch realistische Probleme angewandt (Strahlungsaustausch in Gasen mit innerer Wärmequelle und Wärmeübergang bei laminarer Strömung eines absorbierenden und emittierenden Gases zwischen parallelen Platten).

ПРИМЕНЕНИЕ МОДЕЛЬНЫХ ВЫРАЖЕНИЙ ДЛЯ ПОГЛОЩАТЕЛЬНОЙ СПОСОБНОСТИ В ИНФРАКРАСНОМ ДИАПАЗОНЕ К НЕСЕРОМУ ИЗЛУЧЕНИЮ

Аннотация — Рассмотрены различные зависимости для полной поглощательной способности. Эти зависимости использовались для изучения их влияния на конечные результаты по лучистому переносу в двух физических задачах (лучистый перенос в газах при наличии внутреннего источника тепла и перенос тепла при ламинарном течении поглощающих-излучающих газов между параллельными пластинами).